

# Measurement of helium-3 and deuterium stopping power ratio for negative muons

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**Abstract.** The measurement method and results measuring of the stopping power ratio of helium-3 and deuterium atoms for muons slowed down in the D/<sup>3</sup>He mixture are presented. Measurements were performed at four values of pure <sup>3</sup>He gas target densities,  $\varphi_{\text{He}} = 0.0337, 0.0355, 0.0359, 0.0363$  (normalized to the liquid hydrogen density) and at a density 0.0585 of the D/<sup>3</sup>He mixture. The experiment was carried out at PSI muon beam  $\mu\text{E4}$  with the momentum  $P\mu = 34.0$  MeV/c. The measured value of the mean stopping ratio  $S_{3\text{He}/\text{D}}$  is  $1.66 \pm 0.04$ .

**PACS.** 34.50.Bw Energy loss and stopping power – 36.10.Dr Positronium, muonium, muonic atoms and molecules

## 1 Introduction

Atomic capture of pions and muons stopped in a mixture of helium and hydrogen isotopes has been a subject matter of quite a lot of experimental [1–9] and theoretical investigations [10–20]. The investigation of this process is important for understanding physics of exotic system and also for studying elementary processes occurring when negatively charged particles stop in a material. To separate two processes — atomic capture and transfer of muons from  $\mu$ -atoms of hydrogen isotopes in the course of their de-excitation to helium nuclei — is practically impossible if X-rays are used, as usually, as diagnostics. What one usually observes experimentally is the result of interference of a few processes accompanying the muon capture by atoms of hydrogen and helium isotopes. Therefore, it is quite a problem to extract unambiguous information on the law of initial capture of muons by atoms of the hydrogen (deuterium)–helium mixture components. The earlier made assumption that the probability for direct muon capture by atoms of the mechanical H<sub>2</sub>(D<sub>2</sub>)/M (M is <sup>3,4</sup>He, Ar, Ne...) mixture is proportional to the charge and concentration of each component (Fermi–Teller  $Z$ -law [21]) turned out to be wrong. Experimental data [17] revealed deviation from the Fermi–Teller law. The  $Z$ -law is based on the assumption that the atomic capture probability is proportional to muon energy loss on atoms of the mixture components. Actually, there is no simple relation between the stopping power of a particular type of atom and the probability for muon capture by this atom.

For the binary mixture He/H it is convenient to express the atomic capture probabilities  $W_{\text{He}}, W_{\text{H}}$  by per-atom capture ratio  $A$  (reduced ratio) defined as  $\frac{W_{\text{He}}/C_{\text{He}}}{W_{\text{H}}/C_{\text{H}}}$

$$W_{\text{H}} = \frac{1}{1 + Ac}, \quad W_{\text{He}} = \frac{Ac}{1 + Ac}, \quad (1)$$

where  $c = C_{\text{He}}/C_{\text{H}}$  is the ratio of the helium and hydrogen atomic concentrations.

The authors of reference [1], who measured the capture probabilities of ( $\pi^-$ )-mesons by hydrogen and helium-3 atoms in the He/H mixture, have found that expressions (1) fit well the experimental data when the slowing-down parameter  $S$  is used instead of the capture ratio  $A$

$$W_{\text{H}} = \frac{1}{1 + Sc}, \quad W_{\text{He}} = \frac{Sc}{1 + Sc}. \quad (2)$$

Here  $S = \bar{s}_{\text{He}}/\bar{s}_{\text{H}}$  is the ratio of the averaged stopping powers, where

$$s_i = A_i \left( -\frac{dE}{d\xi} \right)_i, \quad i = \text{He, H}, \quad (3)$$

are per-atom stopping powers of <sup>3</sup>He and H atoms (expressed in MeV cm<sup>2</sup>/atom),  $\xi$  is the mass thickness and  $A_i$  are the atomic masses.

Formulae (2) can also be used for negative muons captured in the H/He mixture (with a proper  $S$ -value) due to similarity of  $\mu^-$  and  $\pi^-$  masses ( $207m_e$  and  $273m_e$ , respectively).

Recently a series of experiments on the study of  $\mu$ -atomic and  $\mu$ -molecular processes in a D/<sup>3</sup>He mixture

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has been carried out at the Paul Scherrer Institute meson factory [22–24]. In the experiments we measured the following characteristics: the nuclear fusion rate in a charge-asymmetrical muon complex ( $d\mu^3\text{He}$ ); the probability for transition of the  $d\mu$  atom from the excited to the ground state ( $q_{1s}$ ); intensities of X-ray radiation of  $\mu\text{He}$  atoms resulting both from muon capture by  $^3\text{He}$  atoms and from transfer of the muon from the  $d\mu$  atom under its de-excitation to the  $^3\text{He}$  nucleus.

For correct interpretation of the above-mentioned experimental data it was necessary to have information on the probability for direct capture of muons by deuterium and helium atoms in the  $\text{D}/^3\text{He}$  mixture. On the one hand, the value of the capture ratio averaged over the data of the papers [1–9],  $A = 1.72 \pm 0.2$ , can be used; on the other hand, an attempt may be made to get experimental information on  $S = S_{\text{He}/\text{D}}$  as a ratio of stopping powers of helium and deuterium atoms in an independent way from an additional experiment. The description of the method and analysis of the results of such measurement of the  $S$ -value for muons as the ratio of stopping powers of helium and deuterium per atom is the aim of this work.

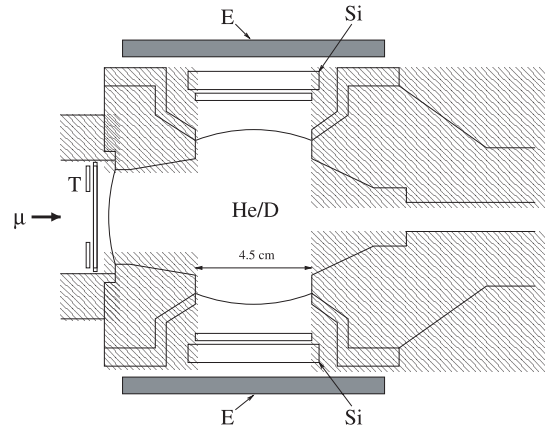
## 2 Measurement method

In our experiment we used two types of gas targets; one was a  $\text{D} + 5\%^3\text{He}$  mixture with the density  $\varphi_{\text{mix}} = 0.0585$  (hereafter all atom number densities  $\varphi$  are normalized to the liquid hydrogen density,  $n_o = 4.25 \times 10^{22} \text{ cm}^{-3}$ ), the other was a target with pure helium-3. A set of different helium densities  $\varphi_{\text{He}}$  was considered. The deuterium-helium mixture and the helium targets were exposed to the same muon beam in order to keep the same initial energy distributions of muons entering the targets. The momentum of the muonic beam was chosen such as to stop all entering muons inside the  $\text{D}/^3\text{He}$  target. In the final stage of the slowing-down muons are captured by the target atoms, form muonic atoms and finally decay via the  $\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$  reaction. Decay electrons are then markers of the stopping events<sup>1</sup>.

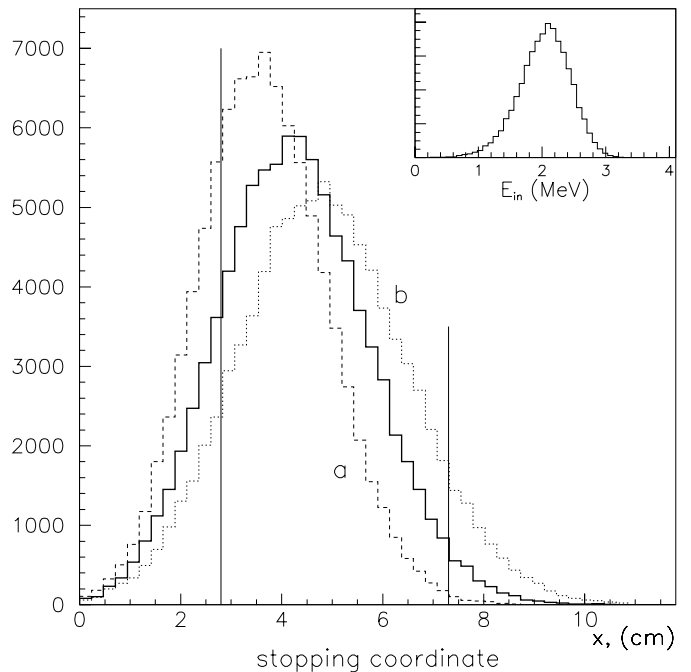
A schematic view of the experimental setup is shown in Figure 1 (see [22,25] and the next section for details).

Varying the gas density in the pure helium target we changed the position of the maximum in the muon stopping distribution along the target length. The example of the Monte Carlo simulations presented in Figure 2 illustrates such a situation. A Monte Carlo stopping code [26] used for obtaining muon stop distributions in the targets include Ziegler parameterization of the stopping powers [27]. Calculations were performed for the 34.0 MeV/c muon beam. Stopping distributions were calculated for three targets: the  $\text{D}+5\%^3\text{He}$  mixture with the density

<sup>1</sup> Because the mean drift of the muonic atoms in the time interval between their formation and the muon decay is small (Monte Carlo calculation gives a value of about 1.5 mm for our experimental conditions) the differences between the spatial distributions of muon stops and the distributions of decay electrons are negligible.



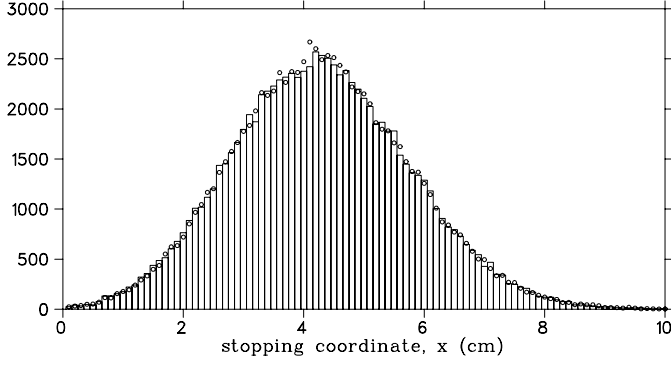
**Fig. 1.** Scheme of the experimental setup. E – electron counters, Si – silicon telescopes.



**Fig. 2.** Stopping distributions calculated for three targets with different densities  $\varphi$ :  $\text{D}/^3\text{He}$  mixture ( $\varphi_{\text{mix}} = 0.0585$ , thick solid line),  $^3\text{He}$  ( $\varphi_{\text{He}} = 0.0320$ , histogram (a)),  $^3\text{He}$  ( $\varphi_{\text{He}} = 0.0380$ , histogram (b)). The beam momentum is 34.0 MeV/c. The energy of incident muons,  $E_{in}$ , is shown in the insert.

$\varphi_{\text{mix}} = 0.0585$  (thick solid line), pure  $^3\text{He}$  with density  $\varphi_{\text{He}} = 0.0380$  (dashed line) and pure  $^3\text{He}$  with the density  $\varphi_{\text{He}} = 0.0330$  (dotted line). Vertical lines show the position of the central (spherical) part of the target; this part is directly seen by the electron counters. The stopping coordinate  $x$  is a distance to the stopping point taken from the entrance window along the beam direction. It is clear that the number of electrons detected by the electron counters will change when the stopping distribution is shifted.

The density of the pure helium target can be chosen such that the number of muon stops detected via decay electrons is the same as in the  $\text{D}_2 + ^3\text{He}$  mixture



**Fig. 3.** Calculated stopping distribution for the  $D/{}^3\text{He}$  target (histogram, solid line) and the equivalent distribution for pure the  ${}^3\text{He}$  target with the density  $\tilde{\varphi}_{\text{He}} = 0.0342$  (open circles).

(we denote this density by  $\tilde{\rho}_{\text{He}}$ ). In such a case spatial distributions of muon stops are equivalent. Such equivalence of the stopping distributions was verified by performing the MC simulations with a set of different  ${}^3\text{He}$  densities and examining the differences by  $\chi^2$ -analysis for both distributions (from  $D/{}^3\text{He}$  and  ${}^3\text{He}$  targets). Figure 3 shows, as an example, the distributions where the minimum of  $\chi^2/df = 0.92$  was achieved for pure helium density  $\tilde{\varphi}_{\text{He}} = 0.0342$ . Similar equivalence of the stopping distributions is obtained in the plane perpendicular to the beam axis. The radial distributions have a Gaussian-like shape with the same 2.1 cm FWHM in both cases (mixture and helium targets). The details of the experimental results discussed later (see points (i) and (ii) in the next section) also justify an assumption of equivalence of spatial distributions of muon stops for a selected He target density.

From the identity of the spatial distributions in both targets (considering that the initial muon energy distributions are also identical) follows equality of the ranges of stopped muons for any initial muon energy  $E_{in}$

$$\int_0^{E_{in}} \frac{1}{(-\frac{dE}{dx})_{\text{He}}} dE = \int_0^{E_{in}} \frac{1}{(-\frac{dE}{dx})_{\text{mix}}} dE. \quad (4)$$

The above equation can be rewritten in terms of the atomic stopping powers  $s_{\text{He}}$ ,  $s_{\text{D}}$

$$\int_0^{E_{in}} \frac{dE}{\tilde{\varphi}_{\text{He}} s_{\text{He}}} = \int_0^{E_{in}} \frac{dE}{\varphi_{\text{mix}} (S^{-1} C_{\text{D}} + C_{\text{He}}) s_{\text{He}}}, \quad (5)$$

where  $S$  is the ratio of stopping powers being the subject of the measurement

$$S = \frac{s_{\text{He}}(E)}{s_{\text{D}}(E)}. \quad (6)$$

In Appendix A we argue how a simple and more useful formula for the mean ratio of the stopping powers can be derived from relations (4), (5) when the behavior of the individual stopping powers of helium-3 and deuterium is taken into account. Such a formula reads

$$S_{\text{He/D}} = \bar{S} = S(\bar{E}) = \frac{C_{\text{D}} \varphi_{\text{mix}}}{\tilde{\varphi}_{\text{He}} - C_{\text{He}} \varphi_{\text{mix}}}, \quad (7)$$

**Table 1.** Conditions of the experiment.  $C_{\text{He}}$  is the atomic concentration of helium,  $N_{\mu}$  is the number of muons that entered the target.

| Run | Target            | Temp. [K] | Pressure [atm] | $\varphi$ [LHD] | $C_{\text{He}}$ [%] | $N_{\mu}$ [ $10^9$ ] |
|-----|-------------------|-----------|----------------|-----------------|---------------------|----------------------|
| 1   | ${}^3\text{He}$   | 32.9      | 6.92           | 0.0363          | 100                 | 1.3625               |
| 2   |                   |           | 6.85           | 0.0359          |                     | 0.7043               |
| 3   |                   |           | 6.78           | 0.0355          |                     | 0.7507               |
| 4   |                   |           | 6.43           | 0.0337          |                     | 0.4136               |
| 5   | $D/{}^3\text{He}$ | 32.8      | 5.11           | 0.0585          | 4.96                | 8.875                |

where  $\bar{E}$  is the average energy of the initial muon energy distribution. The above formula gives the recipe for measurement of the mean ratio of the helium-3 and deuterium atomic stopping powers.

The specific density  $\tilde{\varphi}_{\text{He}}$  (needed for obtaining  $S$  by formula (7)) is experimentally established by measuring the yields of electrons from muon decays in the  $D/{}^3\text{He}$  mixture target and in a set of pure  ${}^3\text{He}$  targets.

### 3 Measurement and results

The experiment was carried out at muon channel  $\mu\text{E4}$  at the Paul Scherrer Institute meson factory. An experimental setup (see Fig. 1) developed for studying the muon-catalysed nuclear fusion reaction  $d\mu{}^3\text{He} \rightarrow \alpha + \mu + p$  (14.6 MeV) [24] was used to measure  $S_{\text{He/D}}$ .

The body of the cryogenic gas target was made of pure Al in the form of a sphere 250 cm<sup>3</sup> in volume. There were five kapton windows 55 to 135  $\mu$  thick in the target body. The entrance window for the muon beam was 45 mm in diameter, its kapton was pressed with a stainless steel flange with a 1-mm-thick gold ring inserted in it. The other four windows were arranged in a circle and were intended for detection of charged products of fusion reaction in the  $d\mu{}^3\text{He}$  reaction and muonic X-rays from  $\mu\text{He}$  atoms. Electrons from the decay of muons stopped in the target were detected by four pairs of plastic scintillator counters (E) installed around the target. The electron data were stored using pre-scalers with a reduction factor of 200.

The experiment included four runs with pure  ${}^3\text{He}$  and one run with the  $D/{}^3\text{He}$  mixture. The experimental conditions are given in Table 1.

Information on the distribution of muon stops in the target volume in experiments with pure  ${}^3\text{He}$  and with the  $D/{}^3\text{He}$  mixture can be gained from analysis of time distributions of muon decay electrons in  ${}^3\text{He}$  and  $D/{}^3\text{He}$  targets

$$\frac{dN_e^{\text{He}}}{dt} = B_{\text{Al}} e^{-\lambda_{\text{Al}} t} + B_{\text{Au}} e^{-\lambda_{\text{Au}} t} + B_{\text{He}} e^{-\lambda_{\text{He}} t} + B, \quad (8)$$

$$\frac{dN_e^{D/{}^3\text{He}}}{dt} = F_{\text{Al}} e^{-\lambda_{\text{Al}} t} + F_{\text{Au}} e^{-\lambda_{\text{Au}} t} + F_{D/{}^3\text{He}} e^{-\lambda_{D/{}^3\text{He}} t} + F, \quad (9)$$

where  $B_{\text{Al}}$ ,  $B_{\text{Au}}$ ,  $B_{\text{AHe}}$ ,  $F_{\text{Al}}$ ,  $F_{\text{Au}}$  and  $F_{D/{}^3\text{He}}$  are normalized amplitudes,  $B$  and  $F$  are the levels of accidental

coincidences, and  $\lambda_{Al}$ ,  $\lambda_{Au}$ ,  $\lambda_{He}$ ,  $\lambda_{D/{}^3He}$  are the rates of muon disappearance in the target wall material and in the target gas.

Measuring the normalized partial amplitudes  $B_{He}$  and  $F_{D/{}^3He}$  and knowing the muon decay electron detection efficiency averaged over the muon energy distribution, we can determine the number of muon stops in  ${}^3He$  and the  $D/{}^3He$  mixture.

Comparing the results of the analysis of the data obtained under different experimental conditions we introduced a quantity  $R$  for convenience.  $R$  is a ratio between the number of electrons from decays of muons stopped in  ${}^3He$  (or in  $D/{}^3He$  mixture) and the number of incident muons

$$R = \frac{N_e}{N_\mu}. \quad (10)$$

The scenario of the experiment was as follows. First, the target was filled with a  $D/{}^3He$  mixture at the density  $\varphi_{mix} = 0.0585$  and the initial muon beam momentum  $P_\mu$  was varied to find its value corresponding to the high value of  $R$ . This value corresponded to the high density of muon stops in the gas at the center of the target. Then the target was filled with pure  ${}^3He$  to the pressure at which the number of stops in the target approximately corresponded to the number of muon stops in the  $D/{}^3He$  mixture or, to be more exact, at which the numbers  $N_e^{D/{}^3He}$  and  $N_e^{He}$  of detected electrons from decays of muons stopped in the gas (in  ${}^3He$  and  $D/{}^3He$  mixture) were rather close<sup>2</sup>.

As was mentioned earlier, the fact of the identity of the muon stop distributions for the  $D/{}^3He$  target at  $\varphi_{mix}$  and the He target at  $\tilde{\varphi}_{He}$  is crucial for our analysis. There are noteworthy points confirming this fact:

- (i) the numbers of muon stops in the entrance ring of Au and the target walls of Al are equal (per incident muon) in both cases;
- (ii) the ratios between the numbers of stops in the target walls and the gas are equal in both cases:

$$N_\mu^{Al}(\tilde{\varphi}_{He})/N_\mu^{He}(\tilde{\varphi}_{He}) = N_\mu^{Al}(\varphi_{mix})/N_\mu^{D/{}^3He}(\varphi_{mix}),$$

$$N_\mu^{Au}(\tilde{\varphi}_{He})/N_\mu^{He}(\tilde{\varphi}_{He}) = N_\mu^{Au}(\varphi_{mix})/N_\mu^{D/{}^3He}(\varphi_{mix}).$$

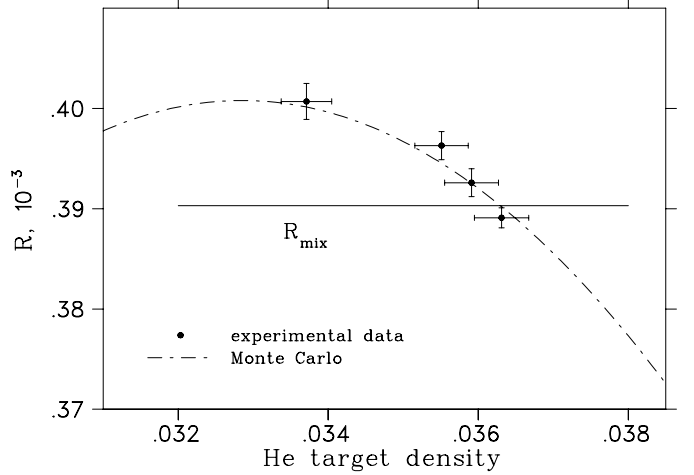
To find out how the ratio  $R$  varies with the  ${}^3He$  density, four runs were carried out with the helium density varying by a few% from run to run (see Tab. 1).

To find the number of muon stops in runs with  ${}^3He$  and the  $D/{}^3He$  mixture, time distributions of muon decay electrons were approximated by expressions (8) and (9) (see Ref. [24] for more detailed description). Table 2 presents the numbers of detected muon decay electrons as well as the ratios  $R = N_e/N_\mu$  measured in runs 1–5 with pure helium and  $D/{}^3He$  mixture. The results of the measurement are also shown in Figure 4, where four experimental points of  $R$  for pure helium versus target density are plotted. The abscissa of the intersection point of  $R(\varphi_{He})$ -dependence with the horizontal line representing the  $R(\varphi_{mix})$  value

<sup>2</sup> The proper  ${}^3He$  pressure in the target was initially chosen by using Monte Carlo stopping code [26].

**Table 2.**  $R$ -ratio measured in runs 1–5.  $N_\mu$  is the number of muons entering the target and  $N_e$  is the number of detected electrons from muon stops in the  ${}^3He$  and  $D/{}^3He$  targets.

| Run | Target     | $\varphi$<br>[LHD] | $N_\mu$<br>[ $10^9$ ] | $N_e$<br>[ $10^6$ ] | $R$<br>[ $10^{-3}$ ] |
|-----|------------|--------------------|-----------------------|---------------------|----------------------|
| 1   | ${}^3He$   | 0.0363             | 1.3625                | 0.5302 (14)         | 0.3891 (10)          |
| 2   |            | 0.0359             | 0.7043                | 0.2765 (10)         | 0.3926 (14)          |
| 3   |            | 0.0355             | 0.7507                | 0.2975 (10)         | 0.3963 (14)          |
| 4   |            | 0.0337             | 0.4136                | 0.1657 (8)          | 0.4007 (18)          |
| 5   | $D/{}^3He$ | 0.0585             | 8.875                 | 3.4635 (35)         | 0.3903 (4)           |



**Fig. 4.**  $R = N_e/N_\mu$  is the ratio between the number of decay electrons detected by the electron counters and the number of incident muons (black circles) for four pure  ${}^3He$  targets used in the experiment (densities:  $\varphi = 0.0337, 0.0355, 0.0359, 0.0363$ ). The horizontal solid line represents the experimental value of  $R$  for the  $D/{}^3He$  target (mixture of deuterium and 5% of  ${}^3He$ ,  $\varphi = 0.0585$ ). The dash-dotted line is the Monte Carlo simulation normalized by the factor  $R_{exp}(\varphi_{mix})/R_{MC}(\varphi_{mix})$ . MC calculations were performed using theoretical helium stopping powers scaled by the factor of 0.96 in order to obtain better description of the experimental points.

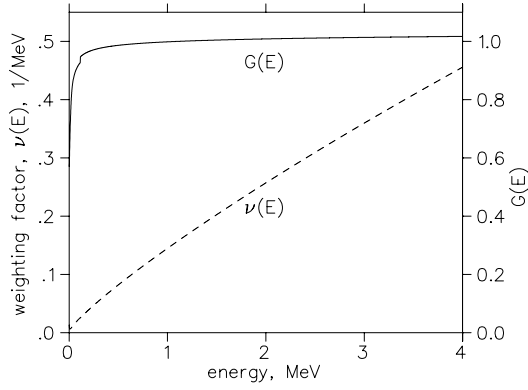
for the  $D/{}^3He$  mixture is just the one to determine the value of the equivalent helium density  $\tilde{\varphi}_{He}$ .

In order to evaluate the character of  $R$ -dependence on  $\varphi_{He}$  the auxiliary Monte Carlo calculations were performed for our experimental conditions. The result of the simulation (dash-dotted line in Fig. 4) shows the non-linearity of the  $R$ -dependence.

From the analysis of the data presented in Table 2 and in Figure 4 the equivalent helium density was found:  $\tilde{\varphi}_{He} = 0.0363 \pm 0.0005$ . According to formula (7), the experimental value of the mean stopping power ratio of helium-3 and deuterium atoms is

$$S_{He/D} = 1.66 \pm 0.04. \quad (11)$$

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**Fig. 5.** The functions  $\nu(E)$  and  $G(E)$  (formulae (12) and (13), respectively).  $G(E)$  is calculated for the densities  $\tilde{\varphi}_{\text{He}} = 0.0342$ ,  $\varphi_{\text{mix}} = 0.0585$ . The weighting function  $\nu(E)$  is shown for  $E_{\text{in}} = 4$  MeV.

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## Appendix A: Mean ratio of stopping powers

Using notations

$$\nu(E) = \frac{1/s_{\text{He}}(E)}{\int_0^{E_{\text{in}}} 1/s_{\text{He}}(E) dE}, \quad (12)$$

$$G(E) = \frac{\tilde{\varphi}_{\text{He}}}{\varphi_{\text{mix}}(S(E)^{-1}C_{\text{D}} + C_{\text{He}})}, \quad (13)$$

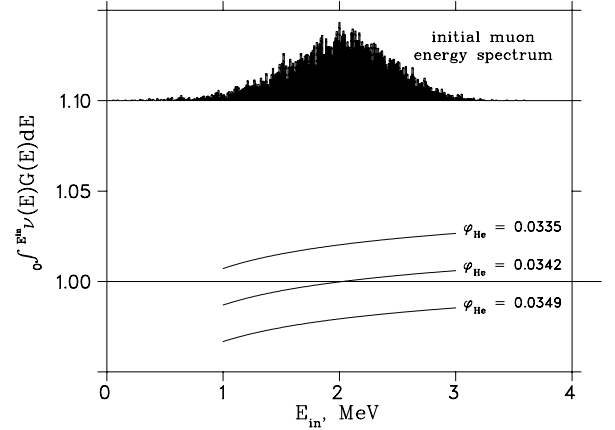
the equality of ranges (5) can be rewritten as

$$\int_0^{E_{\text{in}}} \nu(E) G(E) dE = \bar{G} = 1. \quad (14)$$

Two components of the integrated function in equation (14) depend on the muon energy  $E$  in a quite different manner as is seen in Figure 5.

The weighting function  $\nu(E)$  (normalized to unity) is strongly energy dependent and decreases roughly linearly with decreasing energy.  $G(E)$  (energy dependent via  $S(E)$ ) is, contrary to  $\nu(E)$ , approximately constant in a wide energy region. In Figure 6 the calculated ratio of the muon ranges in  $\text{D}^3\text{He}$  and pure helium-3 targets  $\bar{G} = \int_0^{E_{\text{in}}} \nu(E) G(E) dE$  as a function of the initial muon energy,  $E_{\text{in}}$ , is presented. For given densities  $\varphi_{\text{mix}}$ ,  $\varphi_{\text{He}}$ , this ratio is practically independent of  $E_{\text{in}}$ . It is a consequence of the behaviour of the integrated function (or, in other words, due to similar dependence of deuterium and helium stopping powers on the muon energy). In the energy interval 1.8–2.3 MeV (50% of beam muons belong to this interval) the relative change of  $G$  is 0.4%, and for interval 1.3–2.7 MeV (90% of muons) the respective change is 1%.

The quantity  $\bar{G}$  in equation (14) represents the ratio of the ranges of muons with the initial energy  $E_{\text{in}}$  in  $\text{D}^3\text{He}$



**Fig. 6.** Ratio of muon ranges in  $\text{D}^3\text{He}$  mixture and pure helium-3 targets  $\bar{G} = \int_0^{E_{\text{in}}} \nu(E) G(E) dE$  calculated for different initial muon energies  $E_{\text{in}}$ , for three helium-3 target densities: 0.0335, 0.0342, 0.0349. The real energy spectrum of muons entering the targets is also shown (top).

mixture and in pure  $^3\text{He}$  targets. Basically, equality (14) is fulfilled for a given energy  $E_{\text{in}}$  for especially chosen  $\tilde{\varphi}_{\text{He}}$  (as is seen in Fig. 6). For another energy the other density  $\tilde{\varphi}_{\text{He}}$  should be, in principle, adjusted. But as is seen from Figure 6, such uncertainty in  $\tilde{\varphi}_{\text{He}}$  is very small (less than 1% in the range of our muon energy spectrum) and can be neglected.

In view of the above considerations it is reasonable to use an approximation

$$\bar{G} \approx G(\bar{S}(E)) \approx G(S(\bar{E})), \quad (15)$$

where  $S(\bar{E})$  is the ratio of the atomic stopping powers taken for the average energy of the initial muon spectrum  $\sim 2$  MeV. Then from equations (14) and (15) follows the equality

$$\frac{\tilde{\varphi}_{\text{He}}}{\varphi_{\text{mix}}(\bar{S}^{-1}C_{\text{D}} + C_{\text{He}})} = 1, \quad (16)$$

and finally formula (7).

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